AD-428149

DAVID TAYLOR MODEL BASIN
WASHINGTON D.C.

DEPARTMENT OF THE NAVY

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A METHOD FOR PREDICTING SLAMMING FORCES ON AND RESPONSE OF A SHIP HULL

by

Ralph C. Leibowitz

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

September 1963 Report 1691

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TABLE OF CONTENTS

Page

ABSTRACT 1

INTRODUCTION 1

METHOD OF ATTACK 3

DISCUSSION 4

CONCLUSIONS 5

RECOMMENDATIONS 6

ACKNOWLEDGEMENTS 6

APPENDIX A - DIFFERENTIAL EQUATIONS FOR OBTAINING RESPONSE OF A SHIP TO SLAMMING FORCE . . 7

APPENDIX B - DATA REQUIREMENTS AND METHOD FOR COMPUTING DATA 12

B1 TIME-INDEPENDENT QUANTITIES 12

B2 TIME-DEPENDENT QUANTITIES 13

B2.1 Ship Lines. 13

B2.2 Determination of Half-Breadths c, c', c'' and Corresponding Immersions y', y, y''', and Deadrise Angle β 16

B2.3 Determination of Submerged Area A, Section Area Coefficient η, Added Mass Coefficient C_v, and Added (or Virtual) Mass m 18

B2.4 Determination of Velocity of Advance $\frac{dx}{dt}$, and Downward Relative Velocity $\dot{y}_r$ 29

B2.5 Determination of Forcing Function $\overline{P}_e$ and Rigid-Body Motions y_h, $\dot{y}_h$, y_p, $\dot{y}_p$, ψ, $\dot{\psi}$ 30

B2.6 Determination of Total Mass per Unit Length μ (x, t) (Including an Allowance for Virtual Mass m (x, t), and Damping Force $\tilde{\zeta}$(x, t)) 32

Page

APPENDIX C - COMPARISON OF PRESENT METHOD OF COMPUTATION WITH ALTERNATIVE METHOD GIVEN IN APPENDIX F3 (PAGES 80 - 82) OF REFERENCE 1 33

APPENDIX D - EXTENSION OF ANALYSIS TO IRREGULAR WAVES 35

REFERENCES 37

LIST OF FIGURES

Page

Figure B1 - Method for Bridging Hollow Portions of an Immersed Ship Section with Tangent Lines 20

Figure B2 - Submerged Area for a Ship Section Which Includes a Hollow Portion 21

NOTATION

Symbol	Definition	Dimensions
A	Submerged cross-sectional area	ft^2
$A_0, A_1 \ldots$	Coefficients in ship line function	
$a_1, a_3, a_5, m, n, p, \beta, r_1, r_2, s_1, s_2, \xi$	Parameters used in determining added mass; see Appendix B2.3	
$\bar{a}, \bar{b}, \bar{c}$	Coefficients in ship line function	
$(C_V); (C_V)_{y'}$	Added mass coefficient; added mass coefficient for draft y'	
$\tilde{c}, \tilde{\tilde{c}}$	Damping forces per unit velocity per unit length;	ton-sec/ft^2
c, c', c''	Half-breadth of ship at immersion y', y, and y''', respectively	ft
D	Still-water draft	ft
$\frac{dx}{dt} = \frac{d\xi}{dt}$	Longitudinal velocity	ft/sec
E	Modulus of elasticity in tension and compression	ton-ft^2
EI	Flexural rigidity of hull	ton-ft^2
$f(y)$	Half-breadth corresponding to level y	ft
G	Modulus of elasticity in shear	ton/ft^2
g	Vertical acceleration of hull due to gravity	ft/sec^2
h	Wave height	ft

Symbol	Definition	Dimensions
I	Moment of inertia of submerged cross section about transverse axis in free surface	ft^4
$\bar{I}$	Function of I defined in Appendix B2.3	dimensionless
$I_{\mu z}$	Mass polar moment of inertia per unit length of section of hull Δx long taken about a horizontal axis through its c.g. (including allowance for virtual mass of surrounding water)	$ton\text{-}sec^2$
$\tilde{K}$	Defined in Section B 2.6	
K	Shear flexibility factor; $\dfrac{\Gamma\left(\frac{1}{2}+\frac{\beta}{\pi}\right)\Gamma\left(1-\frac{\beta}{\pi}\right)\cos\beta}{\sqrt{\pi}}$; $\dfrac{t}{\Delta t}$	
KAG	Shear rigidity of hull	ton
M	Bending moment on hull	ft-ton
m	Hydrodynamic added mass per unit length	slugs/ft = $lb\text{-}sec^2/ft^2$; also $\dfrac{ton\text{-}sec^2}{ft^2}$
m_s	Ship mass per unit length	slugs/ft; also $ton\text{-}sec^2/ft^2$
n, $n + 1/2$	Station and midstation numbers, respectively, for ship where stations run 0 (stern), 1, ... N - 1, N (bow) and midstations run 1/2, 1 1/2, ... N - 1/2; N is also distance along normal to surface	ft
$\bar{P}(x, t)$	Total force per unit length acting upon the ship hull	ton/ft
P_e, $\bar{P}_e$	Defined on Pages 70-71 of Reference 1	

Symbol	Definition	Dimensions
t	Time	sec
U	Uniform forward velocity of ship along its heading	ft/sec
u	In general, the horizontal component of particle velocity at any depth in the undisturbed fluid	ft/sec
V	Vertical shearing force acting on hull	ton
V_w	Velocity of wave propagation	ft/sec
v, $\dot{v}$	In general, the vertical component of particle velocity and acceleration, respectively, at any depth	ft/sec and ft/sec^2, respectively
W	Weight of hull per unit length	ton/ft
x	Abscissa or axis of abscissas in a rectangular coordinate system; a coordinate indicating distance from wave crest of point on ship; horizontal displacement of particle at time t; particle coordinate of particle at rest; longitudinal distance measured from stern of ship	ft
$x_{c.g.}$	Distance from stern to center of gravity	ft
(y, z)	Coordinate of ship profile for each section	ft
y	Ordinate or axis of ordinate in a rectangular coordinate system; immersion; particle coordinate of a particle at rest	ft
$\bar{y}$	Lateral deflection of hull	ft
$\dot{y}$	Immersion velocity	ft/sec

Symbol	Definitions	Dimensions
y', y'', y'''	Draft; (See Appendix B of Reference 1)	ft
y_ϵ	Elastic (flexural) component of $\bar{y}$	ft
$y_h, \dot{y}_h$	Displacement and vertical velocity of keel, respectively, due to heave	ft and ft/sec, respectively
y_i	i + 1 water level, i = 0, 1 ... N - 1	ft
$y_p, \dot{y}_p$	Displacement and vertical velocity of keel, respectively, due to pitch	ft and ft/sec, respectively
$\dot{y}_r$	Downward relative velocity of hull with respect to fluid undisturbed by ship (not wave)	ft/sec
y_w	Elevation of wave surface above mean water level	ft
Z, z	Complex variables for the transformed and circle planes, respectively	
β	Deadrise angle	deg
$\gamma, \dot{\gamma}$	Rotation and angular velocity, respectively, of transverse sections with respect to a horizontal axis	radians and radians/sec, respectively
Δt	Time interval or increment	sec
Δx	Length of element, $\ell/20$; increment of length	ft
ζ	Distance of station n forward of position of heave meter	ft
$\eta, \dot{\eta}$	Vertical displacement from rest condition and associated vertical velocity of particle at arbitrary depth y; section area coefficient (dimensionless)	ft and ft/sec, respectively

Symbol	Definition	Dimensions
θ	$\frac{2\pi x}{\lambda}$	radians
θ_n	Value of θ at any station n (equal to $0.1366 \frac{t}{0.1096} + 0.2732\, n - 2.732$)	radians
θ_s	Ship's heading	deg
λ	Wave length	ft
μ	Mass per unit length of hull (including allowance for virtual mass)	ton-sec^2/ft^2
$\xi, \dot{\xi}$	ξ is distance of any station n along keel forward of midlength; horizontal velocity of a particle at arbitrary depth y	ft and ft/sec, respectively
ξ_n	Value of ξ at station n	ft
ρ	Density of sea water	lb-sec^2/ft^4
$\psi, \dot{\psi}$	Pitch angle and angular velocity, respectively	radians or deg and deg/sec, respectively
ω	Angular velocity	radians/sec

ABSTRACT

This report describes a method for obtaining digital computer solutions for the excitation forces on and transient response of a ship subject to (hydrodynamic) slam when certain basic data are obtained by computation rather than by measurement.

The method is based on a theory shown in TMB Report 1511 to be in good agreement with experiment for a particular application.

INTRODUCTION

A theoretical analysis and manual computation of the "slamming" (hydrodynamic) forces* acting on a ship, based upon an experimental knowledge of the rigid-body motions of the ship relative to the wave, was presented in Reference 1.** These forces are considered to be due to the time rate of change of fluid momentum and to buoyancy forces incident to immersion of the hull. In addition, by use of a digital computer, a computation was made of the transient elastic response and associated hull girder stresses of the ship due to the total force exerted by the fluid

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* In this report we are considering the slamming forces associated with immersion of a flared bow rather than bottom impact or side pounding forces. The forces referred to here are the time-varying equivalent or integrated load force on each cross section rather than the detailed load distribution around the cross section.

** References are listed on page 37.

on the ship. A comparison between the theoretical and measured stresses for the Dutch destroyer showed good agreement. This method, therefore, offers promise for evaluating various hull shapes in the early design stage if a practical computational procedure, requiring a minimum of input data, can be established. Specifically, it is desirable to reduce the complexity, expense, time, labor, and errors often involved in making numerical computations for predicting the slamming forces on and the dynamic (elastic and rigid-body) response of a ship girder without a prior experimental knowledge of the rigid-body and flexural motions of a ship relative to the wave.

Accordingly, the objective of this report is to describe a method for obtaining digital computer solutions for the excitation forces on and transient response of a slammed ship when certain basic data are obtained by computation rather than by measurement. Moreover, as shown in Reference 2, those parts of the basic data which involve ship properties may also be computed digitally rather than manually using tabulated quantities systematically obtained in a prescribed fashion directly from ship plans.

A digital computer (IBM 7090) is presently being coded at the Applied Mathematics Laboratory of the David Taylor Model Basin to obtain solutions in accordance with this method; this code includes a digital computer routine for computing the hull parameters.

METHOD OF ATTACK

The following procedure, based upon the theory of Reference 1, is used to solve for the excitation forces on and the response of the ship for an arbitrary transient wave of encounter:

a. The basic finite difference equations of motion used to obtain the damped transient vertical response of a ship to slamming forces, derived in Appendix F of Reference 1, are coded for solution. For convenience of reference, the corresponding differential equations are rewritten in Appendix A.

b. Time-independent ship data required for solution of these equations are furnished to the digital computer; see Appendix B.

c. Time-dependent data (e.g., m, $\frac{dx}{dt}$, $\dot{y}_r$, $\overline{P}_e$, $\tilde{\tilde{c}}$, etc.) required for solution of these equations, are computed by the digital computer from other time-independent data, some additional constants, and several points that describe each ship line[3] (i.e., cross-sectional profile); see Appendix B.

d. The dynamic elastic and rigid-body response of the ship due to slam is represented by the digital computer solution of the finite difference equations corresponding to the differential equations of Appendix A, using data obtained in accordance with Items b and c.

An alternative method of attack which was presented in Reference 1, is compared with this new method in Appendix C.

The application of this method to an arbitrary transient wave of encounter is discussed in Appendix D.

DISCUSSION

In the process of digitally computing the forces on and transient response of a slammed ship, intermediate calculations of added (i.e., virtual) mass and ship properties are performed by means of a digital computer. The digital computation of the added mass, using functions approximating a ship line [3] (i.e., here a cross-sectional profile) and of Landweber's[4, 5] equations is given in Appendix B. Landweber's equations include those of Lewis[6] and Prohaska[7] as a special case. Reference 2 describes the digital computation of the ship properties (i. e., hull parameters) using data tabulations obtained by a pre-established orderly procedure from ship plans. These apparently novel methods for obtaining virtual mass and ship properties should tend to reduce the cost, time, complexity, labor, and errors usually involved in making such computations.

The digital calculations are of special interest because their applicability extends beyond the present problem to other vibration problem areas in which the hull is also treated as a beam. Thus their application pertains to the determination of the normal mode response of a hull,[8] steady-state response of a hull to a sinusoidal driving force,[9] flutter

response of a hull-control surface system subject to hydrodynamic forces on the rudder,[10,11] etc. Such mechanization fits the trend toward routinizing complex calculations which lead to eventual design utility.

CONCLUSIONS

A method has been devised for obtaining digital computer solutions for excitation forces on and transient response of a ship subject to hydrodynamic "slamming" forces when certain basic data are obtained by computation rather than by measurement*. This method reduces the complexity, expense, time, labor, and errors heretofore connected with obtaining such solutions. Consequently, while a ship is still on the drawing board, it now appears possible and practical to use a high-speed and flexible computer to determine the response to slam for several different combinations of parameters that represent different ship-sea characteristics (see Appendix D) and operating conditions.

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* See first footnote on page 1.

RECOMMENDATIONS

It is recommended that this method be used in the early design stages to evaluate the response of various hull shapes* to slamming forces of the type described in this report. Computer results obtained by systematically varying sea states,** ship parameters, ship geometry (hull shape), speed, heading, etc., presented in the form of design curves, should prove helpful in furnishing useful hull design information for direct use by the Preliminary Design Branch of the Bureau of Ships.

ACKNOWLEDGEMENTS

The author is indebted to Mrs. Susan J. Voigt*** and Dr. Elizabeth Cuthill for their assistance in establishing the procedure for computing the hydrodynamic force by means of a digital computer. The author has also profited from discussions on ship line theory with Dr. Feodor Theilheimer.

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* The severity of hull girder stresses incident to bow immersion was strikingly demonstrated by observations and analyses of strains measured on USS ESSEX (CVA9) during a storm passage around Cape Horn; see Reference 12.

** This would require some modification of the method corresponding to the mathematical description of the wave of encounter under consideration; see Appendix D.

*** Mrs. Susan J. Voigt is presently working on a digital computer code, using the equations presented in this report, for computing the slamming forces on and response of a ship's hull. It is planned to issue a separate report on this computer program.

APPENDIX A

DIFFERENTIAL EQUATIONS FOR OBTAINING RESPONSE OF A SHIP TO SLAMMING FORCE

The basic differential equations used to describe the motions of a ship subject to slam, derived in Appendix F of Reference 1, are (see notation): (The corresponding finite difference equations[1] have been coded for the IBM 7090).

$$\mu(x,t)\frac{\partial \dot{y}_\epsilon(x,t)}{\partial t} + c(x,t)\dot{y}_\epsilon(x,t) + m(x,t)\frac{dx(x,t)}{dt}\left[\dot{\gamma}(x,t) - \frac{1}{KAG(x)}\frac{\partial V(x,t)}{\partial t}\right]$$

$$+\frac{\partial V(x,t)}{\partial x} = \bar{P}(x,t) + m(x,t)\frac{dx(x,t)}{dt}\dot{\psi}(t)$$

$$\frac{1}{EI(x)}\frac{\partial M(x,t)}{\partial t} - \frac{\partial \dot{\gamma}(x,t)}{\partial x} = 0$$

$$I_{\mu z}(x)\frac{\partial \dot{\gamma}(x,t)}{\partial t} - \frac{\partial M(x,t)}{\partial x} + V(x,t) = 0$$

$$\frac{1}{KAG}\frac{\partial V(x,t)}{\partial t} + \frac{\partial \dot{y}_\epsilon(x,t)}{\partial x} - \dot{\gamma}(x,t) = -\dot{\psi}(t)$$

$$\frac{\partial y_\epsilon(x,t)}{\partial t} = \dot{y}_\epsilon(x,t)$$

where*

$$\mu(x,t) = m(x,t) + m_s(x)$$

$$\tilde{c}(x,t) = \tilde{\tilde{c}}(x,t) + \frac{\partial m(x,t)}{\partial t} + \frac{\partial m(x,t)}{\partial x} \frac{\partial x(x,t)}{\partial t}$$

$$\bar{P}(x,t) = \bar{P}_e(x,t) - m_s\left(\frac{\partial \dot{y}_p(x,t)}{\partial t} + \frac{d\dot{y}_h(t)}{dt}\right) - \tilde{c}(x,t)\left[\dot{y}_p(x,t) + \dot{y}_h(x,t)\right]$$

$$\bar{P}_e(x,t) = P_e(x,t) - gm_s(x)$$

$$P_e(x,t) = \frac{d}{dt}(m\dot{y}_r) + (g + \dot{v})\rho A$$

and where for present and future use we define:**

t Time; sec

x Distance coordinate along longitudinal centerline axis of ship hull; the independent variable x as used in the equations lies along the same axis as ξ, hence $dx = d\xi$ and $\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}$ Note, however, that this is not the same x used in Figure 1 of Reference 1 and in other parts of that reference where x is normal to the wavefront; ft

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* The damping terms $\tilde{c}$ and $\tilde{\tilde{c}}$ here replace $\bar{c}$ and c in the corresponding equations in Appendix F1 of Reference 1. In contrast to Reference 1, this change in notation now permits a unique definition for the symbol c; i.e., half-breadth of the ship at a particular immersion.

**For more complete definitions, see Reference 8.

$\bar{y}$	Total vertical displacement of point of beam (hull) initially on x-axis, i.e., displacement of point from equilibrium position in still water; ft
y_ϵ	Elastic (flexural) component of $\bar{y}$; ft
$y_h, \dot{y}_h$	Displacement and vertical velocity of keel, respectively, due to heave; ft and ft/sec, respectively
$y_p, \dot{y}_p$	Displacement and vertical velocity of keel, respectively, due to pitch; ft and ft/sec, respectively
$\dot{y}_r$	Downward relative velocity of hull with respect to fluid undisturbed by ship (not wave); ft/sec
γ	Rotation of transverse section about an axis through the section normal to the x-y plane; radians
M	Bending moment; ft-ton
m	Hydrodynamic added mass per unit length; ton-sec^2/ft^2
m_s	Ship mass per unit length; ton-sec^2/ft^2
V	Net shear force in y-direction; ton
μ	Apparent mass per unit length including allowance for virtual mass of surrounding water; ton-sec^2/ft^2
$\tilde{\tilde{c}}$	Damping force per unit velocity per unit length; ton-sec/ft^2
$\bar{P}$	Total force per unit length acting upon ship hull; ton/ft
$I_{\mu z}$	Mass moment of inertia of hull per unit length with respect to an axis normal to the x-y plane; ton-sec^2
EI	Bending rigidity; ton-ft^2

KAG Shear rigidity; ton

ρ Density of sea water; lb-sec^2/ft^4

A Submerged cross sectional area; ft^2

$W = m_s g$ Gravity load per unit length where m_s is the ship mass per unit length; ton/ft

$\psi, \dot{\psi}$ Pitch angle and angular velocity, respectively; radians and radians/sec

v, $\dot{v}$ In general, the vertical component of particle velocity and acceleration, respectively, at any depth; ft/sec and ft/sec^2, respectively

In Equations [A1] through [A5] the following restrictions are imposed:

$$I_{\mu z}(x) \geq 0$$

$$\frac{1}{KAG(x)} \geq 0$$

$$\mu(x,t) > 0$$

$$\frac{1}{EI(x)} > 0$$

The ship is assumed to have free ends. Therefore, the end (boundary) conditions imposed for all times are:

$$V(0,t) \equiv V(\ell,t) \equiv M(0,t) \equiv M(\ell,t) \equiv 0$$

This is equivalent to $V = \frac{d\dot{y}}{dx} = 0$ at $x = 0$, and $x = \ell$ for all times.

The initial conditions required are:

$$y_\epsilon(x,o);\ \dot{y}_\epsilon(x,o);\ \dot{y}(x,o);\ M(x,o);\ V(x,o)$$

The initial conditions actually supplied are:

$$y_\epsilon(x,o);\ \dot{y}_\epsilon(x,o);\ y(x,o)\ \dot{y}(x,o)$$

Then the required $M(x,o)$ and $V(x,o)$ are obtained by using:

$$M(x,o) = EI(x)\ \frac{\partial y(x,o)}{\partial x}$$

$$V(x,o) \approx \frac{\partial M(x,o)}{\partial x}$$

In addition to these end conditions and initial conditions, certain time-independent and time-dependent data are to be furnished as input to the computer (see Appendix B).

The method of computation follows the general procedure given on pages 70-71 of Appendix F1 of Reference 1 except that:

a. Time-dependent quantities used in Equations [A1] through [A5] are now determined by the digital computer in accordance with the methods of Appendix B rather than from data obtained manually from analysis of an oscillographic record of ship motion, as described on page 68 of Reference 1.

b. Certain time-independent quantities (i.e., hull parameters) used in Equations [A1] through [A5] may now be calculated by a digital computer.[2]

APPENDIX B

DATA REQUIREMENTS AND METHOD FOR COMPUTING DATA

To solve the set of equations ([A1] - [A5]) given in Appendix A, the following time-independent and time-dependent data are to be supplied (as input to the computer):*

B1. TIME-INDEPENDENT QUANTITIES

The hull parameters given here may be computed either digitally[2] or manually[8] from ship plans. The time-independent data furnished are:

a. $m_s(x)$

b. $EI(x)$

c. $KAG(x)$

d. $I_{\mu z}(x)$

e. $\tilde{K} = \dfrac{\tilde{\bar{c}}(x, t)}{\mu(x, t)\,\omega}$

f. Initial conditions for rigid-body and flexural motions $\psi(0)$, $\dot{\psi}(0)$, $y_h(0)$, $\dot{y}_h(0)$, $y_p(x, 0)$, $y_\epsilon(x, 0)$, $\dot{y}_\epsilon(x, 0)$, $\gamma(x, 0)$, $\dot{\gamma}(x, 0)$. (Note: ψ is given in radians)

g. Time intervals

h. The following quantities required for the computation of $\bar{\underset{\sim}{P}}_e$ by means of a digital computer:**

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* This section supersedes Section F3 of Appendix F, Reference 1.

** ζ or k required for computing $\bar{P}_e$ by the method of Appendix F3, Reference 1, need not be provided here. Here the program assumes that the heave meter is placed at the center of gravity of the ship, and the computation makes use of that fact. Moreover, (y, z), the coordinates of the ship profile for each section, are now required in place of c (x), the half-breadth required in Appendix F3, Reference 1. This means that the points on the profile can be given at irregular spacings, thus allowing for a better description of the ship line (discussed later).

$\rho,\ U,\ \theta_s,\ g, \lambda,\ h,\ D,\ (y,z)$

and the number of sections or stations and their respective spacings to be used in solving the finite-difference equations.

In solving for $\bar{P}_e$ (see Appendix B2.5), the initial values $\dot{\psi}(0)$ and $\dot{y}_h(0)$ are required (see Item f above).

B2. TIME-DEPENDENT QUANTITIES

Solution of the finite-difference equations[1] by means of the original AML Code (TRC-4) required that time-dependent quantities (*e.g.*, $m, \frac{dx}{dt}, \dot{y}_r, \bar{P}_e, \tilde{\tilde{c}}$, *etc.*) be supplied at several time intervals. However, an extension of TRC-4 has now been devised whereby the time-dependent quantities are computed within each time step. This routine requires only time-independent data, some additional constants, and several points to describe each ship line. The method for representing a ship line[3] and the method for determining the time-dependent quantities are now described.

B2.1. Ship Lines

Initially the revised program fits a series of third-degree polynomials to the set of points describing each ship cross section in order to get a function approximating the ship line.[3] A segment of a cubic is fitted between every two adjacent points so that in a neighborhood of each point, the two adjacent segments coincide (i.e., at the junction point, the function and its first and second derivatives are continuous). The resulting

function may be written explicitly as follows:*

$$f(y) = \bar{a} + \bar{b}y + \bar{c}y^2 + A_0(y-y_0)^3 + A_1(y-y_1)_+^3 + A_2(y-y_2)_+^3 + \dots + A_{n-1}(y-y_{n-1})_+^3$$

where

$y_o = 0$	is the first water level,
$y_1, y_2, y_3 \dots y_{N-1}$	are the second to N-1 water levels used as ordinates in the data points,
$f(y)$	is the corresponding half-breadth at level y, and

$$(y-y_i)_+^3 = 0 \quad \text{for } y \leqq y_i$$
$$= (y-y_i)^3 \quad \text{for } y \geqq y_i \qquad i = 1, 2 \dots N-1$$

Thus all terms that include the factor $(y-y_i)^3$ are to be disregarded whenever that factor is less than or equal to zero.

integrated for the general case:

$$A = \int_0^{\bar{y}} f(y)\,dy = (\bar{a} - \sum_{i=1}^{K} A_i y_i^3)\,\bar{y} + (\bar{b} + 3\sum_{i=1}^{K} A_i y_i^2)\frac{\bar{y}^2}{2} + (\bar{c} - 3\sum_{i=1}^{K} A_i y_i)\frac{\bar{y}^3}{3} + (A_0 + \sum_{i=1}^{K} A_i)\frac{\bar{y}^4}{4} + \frac{1}{4}\sum_{i=0}^{K} A_i y_i^4$$

* The notation in Reference 3 has been altered in the following discussion in order to remain consistent with the coordinate system and notation used in Reference 1. However, the notation was not changed in the actual FORTRAN coding.

where the last term $\frac{1}{4}\sum_{i=0}^{K} A_i (0 - y_i^4)_+ = 0$ for $y_i \leq 0$ and where $K = N - 1$ when all the $(\bar{y} - y_i)$ terms are positive, and $K = N - J$ $(J > 1)$ when all terms $(\bar{y} - y_{N-J+1})$ to $(\bar{y} - y_{N-1})$ are ≤ 0.

In accordance with the methods of Reference 3, the coefficients $\bar{a}$, $\bar{b}$, $\bar{c}$, A_0, A_1, ..., A_{N-1} are determined and stored in the machine ready for use whenever the function is needed. In addition, the number of inflection points in each ship line and their locations are determined for use later to obtain the section area coefficient η for the added mass.

It should be noted here that the function fits the data points very closely, and care should be taken to avoid including unnecessary points as input; often, especially in the case of sections near the midship, inflection points appear when, in reality, there are none. This generally occurs when more than two points are used to describe a nearly straight portion of the ship line. Extreme caution should be exercised in approximating the stern section if it has a very narrow portion below the waterline because the approximation is critical in that region and the function may cross the axis or introduce extraneous humps.

If data points are taken from the final ship specifications given by the naval architect, the above function should be a good approximation; however, if the points are not accurate, fewer points should be used. These can be selected from a plot of all the given points.

B2.2 Determination of Half-Breadths c, c', c'' and Corresponding Immersions y', y, y''' and Deadrise Angle β

When the dynamic problem begins, the immersion y is computed* (see Equation [3] of Reference 1):

$$y = D + y_w - y_h - y_p$$

where (See Appendixes D, E, and F of Reference 1)

$$y_w = \frac{h}{2} \cos \theta_n$$

$$\theta_n = \frac{2\pi}{\lambda} \left[(V_w + U \cos \theta_s) t + \xi_n \cos \theta_s \right]$$

$$V_w = \sqrt{\frac{g\lambda}{2\pi}}$$

$$y_p = (x - x_{c.g.}) \psi$$

$$y_h^{(\nu)} = y_h^{(\nu-1)} + \left(\dot{y}_h^{(\nu)} + \dot{y}_h^{(\nu-1)} \right) \frac{\Delta t'}{2}$$

$$\psi^{(\nu)} = \psi^{(\nu-1)} + \left(\dot{\psi}^{(\nu)} + \dot{\psi}^{(\nu-1)} \right) \frac{\Delta t'}{2}$$

and the superscript in terms of ν refers to time, ν being the current time step described on page 75 of Reference 1; although all the variables are functions of ν, only those equations requiring insertion of ν (or $\nu \pm$ k, k = 1, 2 ...), for the sake of clarity, contain the superscript here, whereas variables in the other equations will be tacitly understood to contain similar superscripts also.

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* See Notation for definition of symbols used here.

Next the downward vertical velocity is found to determine the direction of immersion:*

$$\dot{y} = \dot{y}_w - \dot{y}_h - \dot{y}_p$$

where
$$\dot{y}_w = -\frac{h}{2}\frac{2\pi}{\lambda} V_w \sin\theta_n = -\frac{h}{2}\frac{2\pi}{\lambda} V_w \sin\frac{2\pi x_n}{\lambda}$$

$$= -\frac{h}{2}\frac{2\pi}{\lambda} V_w \sin\frac{2\pi}{\lambda}\left[(V_w + U\cos\theta_s)t + \xi_n\cos\theta_s\right]$$

$$\dot{y}_p = (x - x_{c.g.})\,\dot{\psi}$$

$$\dot{y}_h^{(\nu)} = 2\,\dot{y}_h^{(\nu-1)} - \dot{y}_h^{(\nu-2)}$$

For each section the half-breadth c', corresponding to the computed immersion y, is found simply by solving the function approximating the ship line at that immersion. In addition, calculation of the first derivative yields the slope β of the line at the point in question (see Appendix B of Reference 1).

Szebehely's relation, which accounts for the piled-up water about a body penetrating a water surface, is then employed to determine the actual half-breadth for the computation. If the ship is immersing ($\dot{y} \geq 0$), a value c greater than the half-breadth c' corresponding to the ship line position will be determined; conversely, if the ship is emerging ($\dot{y} < 0$),

---

* In the first computation, the value for $\dot{\psi}$ and $\dot{y}_h$ are extrapolated and then recomputed by other relationships for the variables (see Appendix B2.5).

a value c'' smaller than the half-breadth c' will be determined; and if the immersion has reached a maximum (or a minimum) ($\dot{y} = 0$), the half-breadth will remain unchanged.

The relation between half-breadths given by Szebehely is:

$$\frac{c'}{c''} = \frac{c}{c'} = \frac{\pi}{2} \, \frac{\tan \beta}{\beta} \; \frac{\Gamma\left(\frac{1}{2} + \frac{\beta}{\pi}\right) \, \Gamma\left(1 - \frac{\beta}{\pi}\right) \cos \beta}{\sqrt{\pi}}$$

where c'' < c' < c, c' being the original half-breadth, and tan β is the slope of the ship line at the point (y, c').

If a new half-breadth (c or c'') is determined from c' and the previous equation, it is necessary to find the corresponding draft y' or y''', respectively. This is done with an iteration based on linear interpolation of the ship line function. For this reason either the function must be monotonic and single-valued (i.e., corresponding to a nonbulbous section) or additional information must be furnished to permit determination of which of the two possible values of f (y) is pertinent.

B2.3. Determination of Submerged Area A, Section Area Coefficient η, Added Mass Coefficient C_V and Added (or Virtual) Mass m

Having found the depth of submersion y' or y''' of the ship section, it is possible to calculate the submerged area by the method described in Appendix B2.1 and the section area coefficient η making use of the integrated ship line function by the method now described. The coefficient η, which is used in determining the added mass, is based on a modified section

area. If the section has any concave portions, these must be discounted by drawing a tangent line between convex portions of the ship line and treating the modified section as if it were solid.[7,13] This is done on the machine by an iterative process, starting at the inflection points (see Figure 1). Holding one point A fixed at the point of inflection and moving another point B away from it, the change in the slope of the line connecting the points AB is checked for a change in direction or sign. If the slope is increasing and suddenly begins to decrease, the point of tangency (i.e., the moving point B) is fixed, and then point A is moved away from the inflection point until, again, the change in slope changes sign. This point A is fixed at the second tangency point. However, the new line is no longer tangent to the first tangency point. Therefore, this process is repeated until a line sufficiently tangent to both ends of the concave portion is determined.

If there is only one inflection point in the submerged portion of the ship section, a point on the upper end of the ship line (i.e., point B) is fixed at the waterline and only the lower point A is varied until tangency is obtained. If there are more than two inflection points, one or more tangent lines will be determined in the manner just described.

The area A is then found by adding the area(s) of the trapezoid(s) bounded by the tangent line(s) to the actual area of the remainder of the submerged portion of the ship section (see Figure 2). η is equal to this area A which is the area of one-half the total modified cross section

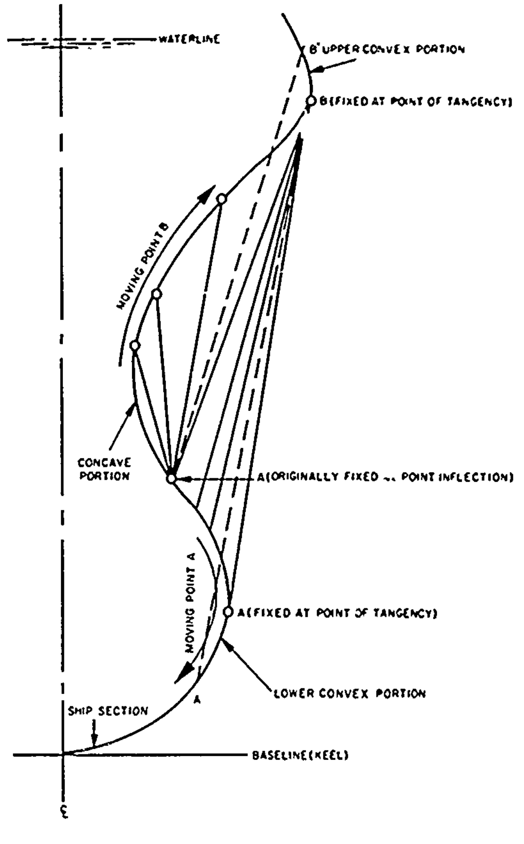

Figure B1 – Method for Bridging Hollow Portions of an Immersed Ship Section with Tangent Lines

First A is fixed at the point of inflection and B moves until the point of tangency of line AB to the upper convex portion is found. Then B is fixed at this point and A moves until the point of tangency of line BA to the lower convex portion is found. The process is then repeated until line AB is sufficiently tangent to both ends of the concave portion, i.e., the upper and lower convex portions. Lines AB' and BA' show the changes in slope of lines AB and BA, respectively.

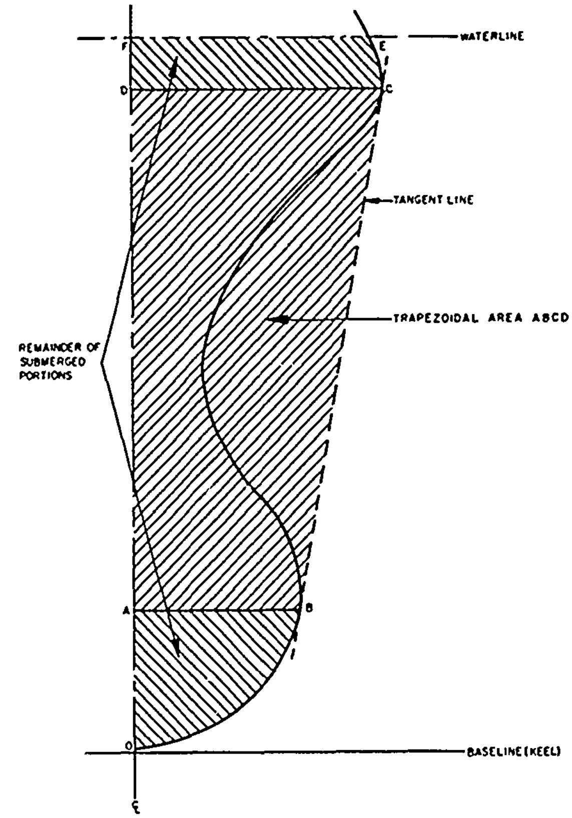

Figure B2 – Submerged Area for a Ship Section which Includes a Hollow Portion

divided by the product of the half breadth and the draft at the water level (i.e., $c \cdot y'$ or $c'' \cdot y'''$ where c, c'' and y', y''' are determined in the manner just described); i.e.,

$$\eta = \frac{A}{c \cdot y'} \text{ or } \frac{A}{c''y'''}$$

The added mass is now determined in the manner described by Lewis,[6] Prohaska,[7,13] and Landweber.[4,5] For a cylindrical hull partially submerged and vibrating vertically, the added mass per unit length is given by

$$m = \frac{\pi}{2}\rho\, c^2\, C_V$$

where ρ is the density of sea water,

c is the local half-breadth at the waterline; in general, c = c or c'' here (see Appendixes B and F3 of Reference 1), and

C_V is a coefficient depending upon the form of the submerged cross section determinable as a function of η, c, and y' (or η, c'', and y''' if the ship is emerging rather than immersing).

The inertial coefficient C_V may be determined by a conformal transformation of the known flow about an elliptic cylinder of approximate ship section. Consider the mathematical transformation used by Landweber and Macagno[4,5]

$$Z = z + \frac{a_1}{z^m} + \frac{a_3}{z^n} + \frac{a_5}{z^p}; \quad a_1, a_3, a_5 \text{ real}$$

in which the indices m, n, etc., are odd numbers for the case of symmetrical sections. This transformation comprehends the transformations with indices $(m, n) = (1, 3); a_5 = 0$, and $(m, n) = (1, 5)\ (1, 7)\ (3, 7); a_5 = 0$ used by Lewis[6] and Prohaska,[7,13] respectively, to develop an analytical procedure for evaluating the virtual mass of a class of *two-dimensional* forms, representative of ship sections, by means of conformal mapping of a circle. Here z is the complex variable of the circle plane and Z is the complex variable of the transformed plane. Different values of a_1, a_3, and a_5 give different sectional forms and corresponding variations in the two-dimensional virtual mass.

For the *three-parameter* family of two-dimensional forms $(a_5 \neq 0)$ with (m, n, p) = (1, 3, 5), Landweber gives the following results (see Equations [17], [16] (alternate form above it), [4], and [5] of Reference 5):

$$C_V = \frac{\left[(1+a_1)^2 + 3a_3^2 + 5a_5^2\right]}{\left(1 + a_1 + a_3 + a_5\right)^2}$$

$$D = \frac{c}{y'} = \frac{1 + a_1 + a_3 + a_5}{1 - a_1 + a_3 - a_5}$$

$$\eta = \frac{A}{2cy'} = \frac{\pi}{4}\,\frac{(1 - a_1^2 - 3a_3^2 - 5a_5^2)}{(1 + a_1 + a_3 + a_5)(1 - a_1 + a_3 - a_5)} = \frac{\pi}{4}\,\frac{(1 - a_1^2 - 3a_3^2 - 5a_5^2)}{(1 + a_3)^2 - (a_1 + a_5)^2}$$

(Only immersion variables c, y' are considered here. Emersion variables c'', y''' are treated similarly.)

Special cases of these results are:

Lewis Forms: $(m, n) = (1, 3)$; $a_5 = 0$

Prohaska Forms: $(m, n) = (1, 5), (1, 7), (3, 7)$; $a_5 = 0$

For the *Lewis forms*, a_1 and a_3 are solved in terms of p and η, which are known (predetermined) quantities from the last two equations ($a_5 = 0$). Thus

$$a_1 = + \frac{3\pi(p^2 - 1) \pm (p - 1)\sqrt{9\pi^2 (p + 1)^2 - 8\pi(p^2\pi + p\pi + \pi + 4p\eta)}}{4(p^2\pi + p\pi + \pi + 4p\eta)}$$

$$a_3 = \frac{1 - p + (p + 1)a}{p - 1}$$

Since a_1 and a_3 each assume two values, it is necessary to establish some restrictions on the parameters. Consider that only the function $z = x + iy$, which lies in the first quadrant, yields useful forms lying within the rectangle of width 2c and depth y'. For this case, the following restrictions given by Landweber must be imposed (see Equation [41] of Reference 4):*

---

* Landweber's $\lambda = \frac{H}{B} \rightarrow \frac{1}{p} = \frac{y'}{c}$

$$\frac{1}{2p} + 1 \leqq \frac{2}{c} \leqq \frac{5}{4p} + 1 ; \frac{1}{p} \leqq 1$$

$$\frac{1}{p} + \frac{1}{2} \leqq \frac{2}{c} \leqq \frac{1}{p} + \frac{5}{4} ; \frac{1}{p} \geqq 1$$

Only one set of values for a_1 and a_3 meet these requirements, hence a unique value for C_V can be determined.*

The coefficients for the Prohaska forms, a_1 and a_5 or a_1 and a_7 or a_3 and a_7, can be solved in a similar manner. These forms as well as the Lewis forms are, however, a special case of the Landweber forms now to be treated.

To solve the *Landweber three-parameter family of forms* for a_1, a_3, and a_5, we first introduce the parameter

$$\bar{I} = \frac{I}{c(y')^3}$$

where I is the moment of inertia of the submerged cross section about the transverse axis in the free surface

$$I = 2 \int_0^{\bar{y}=y'} f(y)\, y^2\, dy$$

I and $\bar{I}$ are known (predetermined) quantities.

* The resulting added mass per unit length m must be divided by 2000 lb and 32 ft/sec^2 in order to have m in terms of ton-sec^2/ft^2, the unit of mass in Reference 1.

Now a_1, a_3, and a_5 are solved in terms of p, η, and $\bar{I}$ (all known quantities) using the equations for p and η given previously and the following equation (see Equation [10] of Reference 5); the details of solution are not given here.* The results, of course, include those of Lewis and Prohaska as special cases.

$$\bar{I} = \frac{\pi\left(\frac{2}{c}\right)^4 p^3}{128} \Big\{ 2\,[\bar{\beta}^3 + \bar{\beta}^2 a_3 + 2\bar{\beta}\,(a_3^2 - a_3 a_5 + a_5^2) - a_3^2 a_5]$$

$$- \bar{\beta}^4 - 2\bar{\beta}^3 a_3 - 2\bar{\beta}^2\,(4a_3^2 - 5a_3 a_5 + 6a_5^2) + 12\bar{\beta} a_3^2 a_5$$

$$- (3a_3^2 + a_5^2)\,(a_3^2 + 5a_5^2) \Big\}$$

here $\bar{\beta} = 1 - a_1$.

To meet the practical requirement that the forms should be within a rectangle of width 2c and depth y', the following conditions and restrictions are imposed:[5**]

Conditions for $\alpha = \frac{2}{c}$:

Lower limit: $r_1 \gtrless r_2$ for $a_5 \lessgtr \frac{\xi}{3}$

Upper limit: $s_1 \gtrless s_2$ for $a_5 \lessgtr \frac{\xi}{27}$

---

* Calculations of the added mass using these relations have been made on the IBM digital computer at the Computer Center, University of California, Berkeley, California.[14] The program for the control of the automatic computations permits selection of any foreseeable number of coefficients—many simple calculations have been done with various combinations of coefficients through a_7.

** α and σ (page 28) are notations in Reference 5 equivalent to notations 2/c and η respectively, in present report.

here:

$$r_1 = \frac{\frac{1}{p} + \frac{1}{2}}{1 + a_5}\ ;\ r_2 = \frac{\frac{1}{2p} + 1}{1 - a_5}\ ;\ s_1 = \frac{\frac{5}{4p} + 1}{1 + 3a_5}\ ;\ s_2 = \frac{\frac{1}{p} + \frac{5}{4}}{1 - 3a_5}$$

$$\xi = \frac{\frac{1}{p} - 1}{\frac{1}{p} + 1}\ ;\ \text{and}\ \left| a_5 \right| < \frac{1}{3}$$

Conditions and Restrictions on a_5:

Conditions	Restrictions
$r_1 \leqq s_1$	$a_5 \leqq \dfrac{\frac{1}{p} + 2}{\frac{7}{p} + 2}$
$r_2 \leqq s_1$	
$r_1 \leqq s_2$	$a_5 \leqq \dfrac{\frac{3}{p}}{\frac{11}{p} + 16}$
$r_2 \leqq s_2$	
	$a_5 \geqq \dfrac{-3}{\frac{16}{p} + 11}$
	$a_5 \geqq -\dfrac{\frac{2}{p} + 1}{\frac{2}{p} + 7}$

Restrictions on $\sigma = \eta$:

$$\sigma(r_1) = \sigma m_1 = \frac{\pi}{32\left(\frac{1}{p}\right)(1 + a_5)^2} \left\{ -3a_5^2 \left[8\left(\frac{1}{p}\right)^2 + 6\left(\frac{1}{p}\right) + 3 \right] \right.$$

$$\left. - 4a_5 \left[2\left(\frac{1}{p}\right)^2 - \frac{1}{p} + 2 \right] + 3 \left[2\left(\frac{1}{p}\right) - 1 \right] \right\}$$

$$\sigma(r_2) = \sigma m_2 = \frac{\pi}{32\left(\frac{1}{p}\right)(1 - a_5)^2} \left\{ -3a_5^2 \left[3\left(\frac{1}{p}\right)^2 + 6\left(\frac{1}{p}\right) + 8 \right] \right.$$

$$\left. + 4a_5 \left[2\left(\frac{1}{p}\right)^2 - \frac{1}{p} + 2 \right] - 3\left(\frac{1}{p}\right)\left(\frac{1}{p} - 2\right) \right\}$$

$$\sigma(s_1) = \sigma M_1 = \frac{\pi \left\{ -9a_5^2 \left[47\left(\frac{1}{p}\right)^2 + 44\left(\frac{1}{p}\right) + 32 \right] + 8a_5 \left[4\left(\frac{1}{p}\right)^2 - 17\left(\frac{1}{p}\right) + 4 \right] + 3\left(\frac{1}{p}\right)\left(\frac{1}{p} + .12\right) \right\}}{128\left(\frac{1}{p}\right)(1 + 3a_5)^2}$$

$$\sigma(s_2) = \sigma M_2 = \frac{\pi \left\{ -9a_5^2 \left[32\left(\frac{1}{p}\right)^2 + 44\left(\frac{1}{p}\right) + 47 \right] + 8a_5 \left[4\left(\frac{1}{p}\right)^2 - 17\left(\frac{1}{p} + 4\right) + 3 \left[12\left(\frac{1}{p} + 1\right) \right] \right] \right\}}{128\left(\frac{1}{p}\right)(1 - 3a_5)^2}$$

To ascertain which forms associated with a particular class of ship (or even sections of a given ship) are preferable — as determined by comparison between the theoretical and experimental response — it is desirable that C_V be computed for the Lewis, Prohaska, and Landweber forms; the latter two for various pertinent confirmations of m, n; and m, n, p, respectively. This would, of course, only require coding of the Landweber forms inasmuch as the other forms are included as special cases. Reference 1 used Figure 24 of Reference 7, corresponding to (m, n) = (1, 5), to compute C_V. The final results for the time-varying stress on the hull girder amidships using values of C_V so computed were in good agreement with experiment (see Figure 23 of Reference 1). Good judgment in the initial choice of the analytical form to be used for a particular ship cross section (which may vary between the extreme U to extreme V in actual form) may be applied by comparing the theoretical values and the experimentally obtained values of C_V or virtual mass for the forms (U, UV, V) and indices (m, n) used in References 7, 13, and 15.

B2.4. Determination of Velocity of Advance $\frac{dx}{dt}$ and Downward Relative Velocity $\dot{y}_r$

The downward relative velocity of the ship, $\dot{y}_r$, is now found (see Equations [2] and [3] and Appendixes D and E, Reference 1, and Notation):

$$\begin{aligned}\dot{y}_r &= \dot{y}_w - \dot{y}_h - \dot{y}_p + (U - u \cos \theta_s) \dot{\psi} \\ &= \dot{y} - \frac{dx}{dt} \psi\end{aligned}$$

where

$$\frac{dx}{dt} = \frac{d\xi}{dt} = -(U - u\cos\theta_s)$$

and

$$\dot{u} = -\frac{h}{2}\ \frac{2\pi}{\lambda}\ V_w \cos\theta_n$$

B2.5. Determination of Forcing Function $\bar{P}_e$ and Rigid-Body Motions y_h, $\dot{y}_h$, y_p, $\dot{y}_p$, ψ, $\dot{\psi}$.

$$\bar{P}_e = P_e - gm_s = \frac{d}{dt}(m\dot{y}_r) + (g + \dot{v})\rho A - gm_s$$

where

$$\dot{v} = \ddot{y}_w = -\frac{h}{2}\left(\frac{2\pi}{\lambda}\right)^2 V_w (V_w + U\cos\theta_s)\cos\theta_n$$

and where

$$\theta_n = \frac{2\pi}{\lambda}\left[(V_w + U\cos\theta_s)\,t + \xi_n \cos\theta_s\right]$$

The first computation of $\bar{P}_e$ may not be very accurate due to its *dependence on extrapolated values* for $\dot{\psi}$ and $\dot{y}_h$. Hence, the program re-computes values for $\dot{\psi}$ and $\dot{y}_h$ as follows: (See pages 68 and 76 of Reference 1.)

$$\dot{y}_h^{(\nu)} = \dot{y}_h^{(\nu-1)} + \frac{\Delta t' \sum_{n=0}^{N-1} \left(\overline{P}_e \, \Delta x\right)_{n+1/2}^{(\nu-1/2)}}{\sum_{n=0}^{N-1} \left(m_s \, \Delta x\right)_{n+1/2}}$$

$$\dot{\psi}^{(\nu)} = \dot{\psi}^{(\nu-1)} + \frac{\Delta t' \sum_{n=0}^{N-1} \left(\overline{P}_e \, \Delta x\right)_{n+1/2}^{(\nu-1/2)} \left(x_{n+1/2} - x_{c.g.}\right)}{\sum_{n=0}^{N-1} \left(m_s \, \Delta x\right)_{n+1/2} \left(x_{n+1/2} - x_{c.g.}\right)^2}$$

where $x_{c.g.}$ is the distance from the center of gravity to the stern and the subscript n + 1/2 refers to the midstation values between Stations n and n + 1.

Other parameters are computed from these new values:

$$y_h^{(\nu)} = y_h^{(\nu-1)} + (\dot{y}_h^{(\nu)} + \dot{y}_h^{(\nu-1)}) \frac{\Delta t'}{2}$$

$$\psi^{(\nu)} = \psi^{(\nu-1)} + \left(\dot{\psi}^{(\nu)} + \dot{\psi}^{(\nu-1)}\right) \frac{\Delta t'}{2}$$

$$\dot{y}_p^{(\nu)} = \dot{\psi}^{(\nu)} (x_{n+1/2} - x_{c.g.})$$

$$y_p^{(\nu)} = y_p^{(\nu-1)} + (\dot{y}_p^{(\nu)} + \dot{y}_p^{(\nu-1)}) \frac{\Delta t'}{2}$$

Subscripts on y_p, $\dot{y}_p$ are tacitly understood to pertain in the actual coding. Then the immersion y' is recomputed from the newer values of its dependent variables and the subsequent computations are repeated. The resulting force $\bar{P}_e$ is compared to the first $\bar{P}_e$. If they are not sufficiently close, then $\dot{y}_h$ and $\dot{\psi}$ are again recomputed and the process is repeated until a satisfactory $\bar{P}_e$ is found for each section.

B2.6. Determination of Total Mass per Unit Length μ (x, t) (Including an Allowance for Virtual Mass m (x, t) and Damping Force $\tilde{\tilde{c}}$ (x, t))

Thus the time-dependent parameters m, $\frac{dx}{dt}$ and $\bar{P}_e$ are used to determine μ (= $m + m_s$) and other variables (e.g., $\bar{P}$) required to find the transient vibration of a ship using the equations in Appendix A. In particular, $\tilde{\tilde{c}}$, the time-dependent damping force per unit velocity per unit length, is found from the expression $\frac{\tilde{\tilde{c}}(x,t)}{\mu(x,t)\omega} = \tilde{K}$. The constant $\tilde{K}$ and modal frequency ω are given (ω can be determined easily by obtaining a normal mode solution[8] using a code devised by the Applied Mathematics Laboratory of the Model Basin or, more roughly, from Schlicks Formula). The value of the constant $\tilde{K}$ depends on the class of ship and is determined in accordance with the methods of Reference 16; $\omega = 1$ if it is desired to use the damping constant as a frequency independent parameter.

The remainder of the program is essentially unchanged from the original TRC-4 used in Reference 1.

APPENDIX C

COMPARISON OF PRESENT METHOD OF COMPUTATION WITH ALTERNATIVE METHOD GIVEN IN APPENDIX F3 (PAGES 80 - 82) OF REFERENCE 1

It is of interest to compare the present method with the alternative method of Reference 1 (pages 80-82).

The present method uses the coordinates to determine a function approximating the ship profile (i.e., a function of half-breadth versus immersion) thereby superseding the procedure described on pages 80-81 of Reference 1 (paragraph a) for determining the half-breadth for a given immersion using a data storage technique. Only one-half the ship profile need be considered, due to symmetry.

The present method uses the ship line function in the dynamic problem to find a value for β by taking the arctangent of the first derivative. This supersedes the procedure described on pages 81-82 of Reference 1 (paragraph b) for determining β by interpolation.

The present method obtains the submerged area A by integration rather than by using an approximating sum as described on page 82 of Reference 1 (paragraph c).

In accordance with the present method, the added mass is computed at the time and immersion when it is needed and not prior to the time-dependent calculations as described on page 82 of Reference 1 (paragraphs d and e).

The computation of the immersion and the hydrodynamic force, etc., is described in Appendix B of this report.

APPENDIX D

EXTENSION OF ANALYSIS TO IRREGULAR WAVES

The wave profile treated in this analysis is a regular wave; see page 51 and Appendix D of Reference 1. Such simple waves can, however, serve as a basis for the synthesis of more complex waves.[17] For example, an irregular wave moving in one direction, with wave crests parallel, may be treated as a linear summation of regular sine waves of different wave lengths and periods. This representation can be extended by adding a different constant phase angle to each component wave. Thus, the treatment given in this report for a regular wave can be extended tc include the terms necessary for a deterministic representation of an irregular wave (i.e., arbitrary wave of encounter for a unidirectional sea).

An irregular wave can also be characterized from a statistical point of view. Thus an element of probability may be introduced into the description of the sea by statistically distributing the phase angles in accordance with a certain distribution function so that all phase angles are equally probable.[17] For this case the spectral density function[17] represents the manner in which the energy of the composite wave is distributed over the frequency, thereby fully characterizing the nondeterministic irregular wave of this type.

The sea waves may, however, exhibit an irregularity in two directions. These irregular waves may be synthesized from component

sine waves assumed to move in any direction. The spectral density function now represents the manner in which the energy of the composite waves is distributed in both the frequency interval and the direction interval. Extension of the treatment given in this report for a regular wave may yield a practical procedure for treating nondeterministic (statistical or probability) representations of irregular waves.

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A METHOD FOR PREDICTING SLAMMING FORCES ON AND RESPONSE OF A SHIP HULL, by Ralph C. Leibowitz. Sep 1963. ix, 42p. illus., graphs, refs. UNCLASSIFIED

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The method is based on a theory shown in TMB Report 1511 to be in good agreement with experiment for a particular application.

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2. Ship hulls--Slamming--Mathematical analysis
3. Ship hulls--Rigid body motions--Mathematical analysis
4. Ship hulls--Transients--Mathematical analysis
5. Ship hulls--Vibration--Mathematical analysis
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